Supplier and Buyer Driven Channels in a Two-Stage Supply Chain

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ABSTRACT

We explore the impact of power structure on price, sensitivity of market price, and profits in a two-stage supply chain with single product, supplier and buyer, and a price sensitive market. We develop and analyze the case where the supplier has dominant bargaining power and the case where the buyer has dominant bargaining power. We consider a pricing scheme for the buyer that involves both a multiplier and a markup. We show that it is optimal for the buyer to set the markup to zero and use only a multiplier. We also show that the market price and its sensitivity are higher when operational costs (namely distribution and inventory) exist. We observe that the sensitivity of the market price increases non-linearly as the wholesale price increases, and derive a lower bound for it. Through experimental analysis, we show that marginal impact of increasing shipment cost and carrying charge (interest rate) on prices and profits are decreasing in both cases. Finally, we show that there exist problem instances where the buyer may prefer supplier-driven case to markup-only buyer-driven and similarly problem instances where the supplier may prefer markup-only buyer-driven case to supplier-driven.

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1 Introduction

An interesting issue that has arisen in recent years in the area of supply chain management is how decisions are effected by the bargaining power along the channel. For example, with the advent of the internet in recent years, customers have access to much more information including price, quality and service features of several potential suppliers. This information has in many cases increased their position to acquire goods and services. On the other hand, if the number of suppliers is limited, then clearly a supplier's position is increased.

In this paper, we explore the impact of bargaining power structure on price, sensitivity of market price, and profits in a two-stage supply chain. We examine the case when a single product is shipped from a supplier to a buyer at a wholesale price and then sold to a price-sensitive market. Operational costs (namely distribution and inventory) are included in the analysis.

Throughout this paper we will refer to the supplier as *he* and the buyer as *she*. Two models arise when either the supplier or buyer has dominant bargaining power in the supply channel, similar to the economics literature where a dominant (or leader) firm moves first and a subordinate (or follower) firm moves second (Gibbons [**?**], Stackelberg [**?**]).

The *supplier-driven channel* occurs when the supplier has dominant bargaining power and the *buyer-driven channel* occurs when the buyer has dominant bargaining power. Even though buyer-driven models are encountered less frequently in literature (compared to supplier-driven models), there is practical motivation: Messinger & Narasimhan [**?**] provide an interesting discussion on how the bargaining power has shifted to the retailer (buyer) in the grocery channel.

The organization of the paper is as follows: In Section 2 a brief literature review is provided. In Section 3 we describe the assumptions of our analysis. The supplier-driven and buyer-driven models are then developed in Section 4. An example is presented in Section 5, including a comparison to the coordinated case where total net profit throughout the supply chain is maximized. An analytical comparison of supplier-driven and markup-only buyer-driven cases is given in Section 6. We also show that there are cases under which a supplier will prefer the buyer-driven channel and where a buyer will prefer the supplier-driven channel. Conclusions and future research are described in Section 7.

2 Literature Review

A significant amount of research has been done in the area of supply chain coordination. Much of it has focused on minimizing the inventory holding and setup costs at different nodes of the supply chain network. The classic Clark & Scarf [?] and Federgruen [?] are examples. Traditionally, this type of research has assumed that the task of designing and planning the operations is carried out by a central planner. However, the increased structural complexity and the difficulty of obtaining and communicating all the information scattered throughout the different units of the supply chain is a major block in applying central planning. Research along decentralized control has been done by a number of people and good reviews are found in Whang [?], Sarmiento & Nagi [?], Erenguc et. al. [?], and Stock et. al. [?].

Although a significant portion of the coordination literature assumes vertical integration, some recent research focuses on contractual agreements that enable coordination between independently operated units. Tsay et. al. [?] present a comprehensive review on contract-based supply chain research. Jeuland & Shugan [?] present an early treatment of coordination issues in a distribution channel. Porteus [?] establishes a framework for studying tradeoffs between the investment costs needed to reduce the setup cost and the operating costs identified in the EOQ. He also addresses the joint selection of the setup cost and market price, comparable to Section 4.1 of our research. Abad [?] formulates the coordination problem as a fixed-threat bargaining game, characterizes Pareto efficient solutions and the Nash bargaining solution and proposes pricing schedules for the supplier. Weng [?] also focuses on role and limitation of quantity discounts in channel coordination and shows that quantity discounts alone are not sufficient to guarantee joint profit maximization under a model where both the market demand is decreasing in prices and the operating cost depends on order quantities.

Ingene & Parry [?] investigate a model with fixed and variable costs at the two stages and establish the existence of a menu of two-part tariffs that mimic all results of a vertically integrated system. Wang & Wu [?] consider a similar model and propose a policy that is superior for the supplier when there are many different buyers. Other examples of related research include McGuire & Staelin [?] and Moorthy [?].

An extensive body of literature focuses on optimizing two-stage supply chains with stochastic demands. Cachon [?] and Cachon & Zipkin [?] extend this literature by developing game-theoretic models for the competitive cases of continuous review and periodic review models. Moses & Seshadri [?] consider a periodic review model with lost sales. Netessine & Rudi [?] develop and analyze models of interaction between a supplier (wholesaler) and a single buyer (retailer) for “drop-shipping” supply chains.

In this paper we try to extend the current literature by considering operating costs explicitly along with different power structures of the market channel.

3 Model Assumptions

Figure 1 illustrates the flow of products, information and funds in our models. The product is shipped from the supplier to the buyer at wholesale price t [\$/unit], and is sold to the market by the buyer at market price p [\$/unit]. The deterministic market demand $\mu[p]$ is linear in p: $\mu[p] = m(a-p), \quad m = \frac{d}{b}, \quad a-b \le p \le a, \quad 0 < b \le a, \quad d > 0$. The buyer operates under a simple deterministic EOQ model. She places an order to the supplier for a shipment of size q [units/order], τ [years] before she runs out of her stock. The carrying charge (interest rate) that the buyer uses for calculating cost of in-site and in-transit inventory is r [%/year]. The buyer has a *reservation net profit* R_B, and she will not participate in the channel if her net profit is less than R_B. The supplier does not have any operational costs, but incurs a unit variable cost c independent of any decision variables. This would occur when the supplier is functionally organized and the sales department acts independently of operations (which is typical for large firms).

The profit function of the supplier is given by:

$$\pi_S[t] = (t-c)m(a-p) \tag{1}$$

The supplier's wholesale price t is freight on board (f.o.b.) origin; that is, the buyer makes the payment and then takes responsibility for the product at the origin by bearing the shipment and material handling costs for the shipments to her facility. The buyer pays the carrier at the time the shipment reaches her facility. The shipment takes a deterministic time τ [years] to arrive at the buyer's facility and costs k [\$/order]. The transit time and the cost of an order are independent of the order quantity q [units/order] and the carrier always has sufficient capacity. The cost of an order is incurred at the moment the shipment arrives at the buyer's facility.

The value of the product on-site is accounted as t [\$/unit]. Assuming the value and the holding cost of product dependent on t (rather than a fixed value) makes the profit function nonconcave at certain ranges. Thus, our analysis will yield the type of results related with the regions of concavity and nonconcavity, similar to Porteus [?] and Rosenblatt & Lee [?].

The buyer's decision variables are the market price p and the order quantity q. The profit function of the buyer is given by:

$$\pi_B[p, q] = (p - t)\mu[p] - k\mu[p]/q - qrt/2 - \mu[p]\tau rt \tag{2}$$

The approach of considering fixed unit costs at a certain stage and costs as a function of operational costs at another stage of the supply chain is similar to the analysis of Ford Customer Service Division done by Goentzel [?]. He focused on the customer allocation problem where the cost per unit product at a warehouse that *faces* the customers is fixed, but the cost of routing to a cluster of customers is depends on the choice of customers.

The assumption regarding the channel structure in this paper bears discussion. Monahan [?] notes that single supplier single buyer channels are often invisible to the public, and lists examples where such a relationship may exist:

- "Small, closely-held or privately owned companies, producing exclusively for other larger manufacturers or distributors;
- Common job-shops, supplying customized products for an individual buyer;
- Manufacturing arms or divisions of independently run parent companies."

These examples define a very narrow scope of the economy, and it is true that most of the economic markets are characterized by oligopoly or perfect competition. On the other hand, Stuckey & White [?] explain how site specificity, technical specificity and human capital specificity may create bilateral monopoly. They describe many industries, including mining, ready-mix concrete and auto assembly, that operate as bilateral monopolies.

Single-supplier and single-buyer relationships are common also due to benefits of long-term partnership. Some of these benefits are as follows (Tsay et. al [?]):

- Reduced ordering costs (i.e. reduced ordering overhead, due to established relationship)
- Additional efforts towards compatibility of information systems
- Additional information sharing
- Collaborative product design/redesign
- Process improvement, quality benefits
- Agreement on standards on lead-times and quality measures

Therefore a significant portion of firms in the economy can benefit by such a relationship.

We assume that the parties are interested in a long-term relationship and are using a contract to enforce commitment to the relationship. Even though the dominant party may deviate, we assume that s/he does not do so, since this might have associated costs.

4 Two-Stage Supply Chain Models

Both the supplier-driven and buyer-driven two-stage supply chain models are developed in this Section. Before developing these models, we first discuss the buyer's decision problem given the wholesale price t.

4.1 Buyer's Decision Problem When t is Given

In this section we investigate the decision problem of the buyer, whose objective is to find the optimal market price p_0^*, given the wholesale price t. As mentioned previously, this problem was studied in detail in[**?**], and we summarize and extend the results here.

By setting $\frac{d\pi_B[p,q]}{dq} = 0$ and $\frac{d\pi_B[p,q]}{dp} = 0$ and solving for q and p, we obtain:

$$\begin{aligned} q &= \sqrt{\frac{2km(a-p)}{rt}} && (3) \\ p &= w/2 + k/2q && (4) \end{aligned}$$

where $w = a + (1 + r\tau)t$.

The extreme points of $\pi_B[p,q]$ solve these equations. When these equations are solved simultaneously, we obtain a cubic polynomial equation:

$$\Phi(p) = p^3 + \Phi_2 p^2 + \Phi_1 p + \Phi_0 = 0 \tag{5}$$

where

$$\begin{aligned} \Phi_2 &= -(a+w), \\ \Phi_1 &= ((4a+w)w)/4, \\ \Phi_0 &= -aw^2/4 + krt/(8m) \end{aligned}$$

We use $\pi_B[p]$ to denote the net profit as a function of p alone: $\pi_B[p] = \pi_B[p, q^*[p]]$. Substituting the expression for q from (3) into the net profit function $\pi_B[p]$ yields:

$$\pi_B[p] = \begin{cases} m(a-p)p - (1+r\tau)m(a-p)t - \psi\sqrt{(a-p)t} & \text{if } a-b \leq p < a \\ 0 & \text{if } p = a \end{cases} \tag{6}$$

where $\psi = \sqrt{2krm}$.

Notice that the profit function is defined as 0 when $p = a$ since a discontinuity would otherwise arise. We now define (p_0^*, q_0^*) as the optimal (p, q) pair when the constraint $a - b \leq p$ in the definition of the linear demand function is taken into account. $\pi_B^*[p_0^*, q_0^*] = \pi_B^*[p_0^*]$ is the optimal solution value:

Definition 1 *Let* $p_0^* = argmax_{\{a-b \leq p \leq a\}} \pi_B[p]$, $q_0^* = q[p_0^*]$ *and* $\pi_B^*[p_0^*, q_0^*] = max_{\{a-b \leq p \leq a\}} \pi_B[p]$.

The regions where $\pi_B[p]$ is concave and convex can be found by investigating the second order derivate of $\pi_B[p]$:

$$\frac{d^2\pi_B[p]}{dp^2} = -2m + \frac{\psi t^2 \left((a-p)t\right)^{-3/2}}{4}$$

which gives us the following:

Theorem 1 $\pi_B[p]$ *is concave when* $p \leq \tilde{p}$ *and convex when* $p > \tilde{p}$*, where* $\tilde{p} = a - \left(\frac{\psi\sqrt{t}}{8m}\right)^{2/3}$.

Now let us characterize the cubic equation $\Phi(p) = 0$, when it has three roots:

Theorem 2 *If* $t \leq \bar{\bar{t}}$ *then* $\Phi(p) = 0$ *has three real roots* $p_{0,1}$, $p_{0,2}$ *and* $p_{0,3}$. *Let* $p_0 = argmax_{\{p=p_{0,1},p_{0,2},p_{0,3}\}} \pi_B[p]$. $p_0 < \tilde{p}$. $p_{0,min} = \min\{p_{0,1}, p_{0,2}, p_{0,3}\} < \frac{w}{2}$. $p_0 = \min\{\,\{p_{0,1}, p_{0,2}, p_{0,3}\} \backslash \{p_{0,min}\}\,\}$.

Proof: Given in A.1. □

Theorem 2 tells us that among the three roots of (5), the middle one is the one that maximizes $\pi_B[p]$.

4.2 Supplier-driven Channel

Let us assume that the supplier has dominant bargaining power and has the freedom to decide on any t value that maximizes his net profit with no consideration for the buyer. The buyer reacts to the wholesale price t declared by the supplier by selecting her optimal price $p_0^*[t]$ (and the corresponding $q_0^*[t]$) that maximizes her net profit given t.

Since the supplier knows the cost structure and the decision model of the buyer, he also knows the reaction $p_0[t]$ of the buyer to t. Since $p_0[t]$ determines $p_0^*[t]$ as described earlier, we will present some of our results in terms of $p_0[t]$.

We define the *sensitivity* of the buyer's optimal price $\xi[t, p_0[t]]$ as follows:

Definition 2 *Let $\xi[t, p_0[t]]$ denote the sensitivity of the buyer's optimal price $p_0[t]$ with respect to the supplier's wholesale price t. $\xi[t, p_0[t]]$ is the ratio of marginal change in $p_0[t]$ to marginal change in t at the point $(t, p_0[t])$.*

An expression for $\xi[t, p_0[t]]$ is provided in the following Theorem:

Theorem 3 $\xi[t, p_0[t]] = \frac{\xi_0[t, p_0[t]]}{\xi_1[t, p_0[t]]}$*, where*

$$\xi_0[t, p] = (1 + r\tau)(p^2 - (2a + w)p/2 + aw/2) - kr/(8m) \tag{7}$$

where $w = a + (1 + r\tau)t$, and

$$\xi_1[t, p] = 3p^2 + 2\Phi_2 p + \Phi_1. \tag{8}$$

Proof: Given in Appendix A.2. □

The following Theorem gives a lower bound for $\xi[t, p_0[t]]$ that is independent of any parameters and decision variables:

Theorem 4 $\xi[t, p_0[t]] > 1/2$.

Proof: Given in Appendix A.3. □

An instance that achieves this lower bound is one that has all the logistics related costs equal to zero ($k = 0, r = 0, \tau = 0$). This instance is equivalent to the classic *bilateral monopoly* model in the Economics literature (Spengler [**?**], Tirole [**?**]), where the sensitivity of market price to wholesale price is always 1/2.

The optimal wholesale price for Model 3 can be determined by taking the partial derivative of $\pi_S[t]$ with respect to t and setting it equal to zero:

$$\frac{a - p}{t - c} = \xi[t, p_0[t]] \tag{9}$$

We will refer to solution of the above equation as t_2. Since (9) does not yield a closed form expression for t_2, one has to resort to numerical methods for solving the equation. The following Theorem, with proof given in Appendix A.4, guarantees that the numerical solution would indeed be the global optimum:

Theorem 5 $\pi_S[t]$ *is concave in t.*

4.3 Buyer-driven Channel

In this case the buyer takes an active role and declares a nonnegative price multiplier α and a nonnegative markup β and states that she will set $p = \alpha t + \beta$. The supplier reacts by choosing the t that maximizes his net profit given the α and β declared by the buyer. The buyer has complete knowledge of the reaction wholesale price $t_3[\alpha, \beta]$ that the supplier will respond with to her declared α and β. She chooses her (α, β) so as to maximize her net profit $P_B[\alpha, \beta]$, given $t_3[\alpha, \beta]$. We assume that the supplier will participate in the supply chain channel as long as his profit is nonnegative.

One interesting result is that the buyer would use only a multiplier under these conditions:

Theorem 6 $\beta^* = 0$.

Proof: The supplier's profit function is $\pi_S = (t - c)m(a - p)$. The optimal t value is found by taking the partial derivative of $\pi_S[t]$ with respect to t and setting equal to zero.

$$\begin{aligned} \frac{\partial \pi_S[t]}{\partial t} = m\left((a-p) - \frac{\partial p}{\partial t}(t-c)\right) &= 0 \\ t &= \frac{a-p}{\alpha} + c \end{aligned} \tag{10}$$

Assuming that p is fixed, α and β can be calculated so as to obtain p once t is determined. Therefore, the selection of t determines α and β. Equation (10) shows that the supplier would select a smaller t as α increases, all other things being fixed. The largest α can be obtained by setting $\beta = 0$. Since this fact holds for any fixed p value, $\beta^* = 0$. $\square$

Since the optimal markup is equal to zero ($\beta^* = 0$) then $P(\alpha, \beta)$ can be optimized with respect to α alone to find α^*. By differentiating the supplier's profit function with respect to t and setting it to 0, we get the supplier's optimal wholesale price $t_3[\alpha, \beta] = \frac{a+\alpha c-\beta}{2\alpha}$. The corresponding market price $p_3[\alpha, \beta] = \frac{a+\alpha c+\beta}{2}$ is then found by substituting into the market demand function.

The terms $t_3[\alpha, \beta]$ and $t_3[\alpha, \beta]$ can be substituted into (6) to obtain the buyer's net profit function $P_B[\alpha, \beta]$ in the buyer-driven channel.

$P_B[\alpha, \beta]$ cannot be solved analytically to find (α^*, β^*). On the other hand, Theorem 6 tells us that the buyer will always set $\beta^* = 0$ at the optimal, and will decide only on α. The first order condition $\frac{\partial P_B[\alpha,0]}{\partial \alpha} = 0$ is not analytically solvable. Yet, in the following Theorem, we show concavity of $P_B[\alpha, \beta]$ in α, suggesting that a simple search algorithm would enable us to find α^* which is globally optimal:

Theorem 7 *$P_B[\alpha, 0]$ is concave in α.*

Proof:

The expression in (6) is composed of three components, the first is a function in p multiplied by the positive constant m, the latter components are functions of p and t multiplied by negative constants. We will prove that $f_1[\alpha] = (a - p_3[\alpha, 0])p_3[\alpha, 0]$ is concave, and $f_2[\alpha] = (a - p_3[\alpha, 0])t$ and $f_3[\alpha] = \sqrt{f_2[\alpha]}$ are convex, which will prove the concavity of $P_B[\alpha, 0]$.

First we focus on $f_1[\alpha] = (a - \frac{a+\alpha c}{2})(\frac{a+\alpha c}{2}) = \frac{a^2-\alpha^2c^2}{4}$. Since $df_1^2/d\alpha^2 = -c^2/2 < 0$, f_1 is concave in α.

Next we focus on $f_2[\alpha] = (\frac{a-\alpha c}{2})(\frac{a+\alpha c}{2\alpha}) = \frac{a^2-c^2\alpha^2}{4\alpha}$. Since $d^2f_2/d\alpha^2 = \frac{a^2}{2\alpha^3} > 0$, f_2 is convex in α.

Finally we focus on $f_3 = \sqrt{\frac{a^2-c^2\alpha^2}{4\alpha}}$. The second derivative is $d^2f_3/d\alpha^2 = \frac{3a^4-6a^2c^2\alpha^2-c^4\alpha^4}{8\alpha^3(a^2-c^2\alpha^2)\sqrt{\frac{a^2}{\alpha}-c^2\alpha}}$. Since the denominator is the product of $8\alpha^2$, $(a^2 - c^2\alpha^2)$, and a square-root expression, it is always positive. The numerator can be expressed as $(2a^4 - 2a^2c^2\alpha^2) + (a^4 - 2a^2c^2\alpha^2 - c^4\alpha^4)$, which can further be expressed as $2a^2(a^2 - c^2\alpha^2) + (a^2 - c^2\alpha^2)^2$. Since both of the terms in this expression are positive, we have $\frac{d^2f_3}{d\alpha^2} > 0$, and thus f_3 is convex.

Having shown the concavity of its three components, we conclude that $P_B[\alpha, 0]$ is concave. □

5 An Example Channel Setting

Consider a supply chain channel where $c = 5.600, a = 12.800, k = 480, \tau = 0.010, r = 0.100, m = 2180, b = 12.800, R_B = 0$. Figure 2 shows the profits/costs related with the buyer when the supplier sets the wholesale price as $t = 11.000$.

In Figure 2, $w/2, p_0$ and $\tilde{p} = 12.577$ are indicated with dashed vertical lines. If there were no logistics related costs, the buyer would set her price to $w/2 = 11.906$. The optimal price when the logistics costs are considered is greater: $p_0^* = p_0 = 12.116$. The optimal profit of the buyer is $\pi_B^*[p_0^*] = \pi_B[12.112] = 392.848$. As long as $a - b$ is less than 12.116, the buyer would clearly keep pricing at $p_0^* = p_0 = 12.116$. If $a - b$ were greater than 12.116, the buyer would see if setting $p = a - b$ would bring any profits or not, and if $\pi_B[a - b] \geq R_B$ she would set $p_0^* = a - b$.

In order to compare the buyer-driven and supplier-driven models to "optimal", we use the coordinated net profit function $\pi_C[p, q]$ can be defined by:

$$\pi_C[p, q] = (p - c)\mu[p] - k\mu[p]/q - qrc/2 - \mu[p]\tau rc \tag{11}$$

This function is similar to $\pi_B[p,q]$ in (2), but there are some differences. Here, the profit per unit is $(p-c)$, as opposed to profit per unit $(t-c)$ of $\pi_B[p,q]$. The value of the product on-site and in transit is accounted as c [\$/unit] in $\pi_C[p,q]$, as opposed to t [\$/unit] of $\pi_B[p,q]$.

Cost c reflects the true value of the product, rather than the artificially created wholesale price t. A central planner would use the true value of the product in calculating the logistics costs. Notice that t does not appear in this cost function at all.

Analysis of the coordinated channel, where $t = c = 5.600$, tells us that the optimal market price is $p_0^* = p_0 = 9.269$. The optimal profit of the buyer is $\pi_B^*[p_0^*] = \pi_B[9.269] = 26165.053$.

Next we consider the supplier-driven model, and investigate the best pricing policies of the supplier. We are implicitly assuming that the buyer would prefer to operate at exactly zero profit, rather than destroy the channel. We focus on the supplier's profit maximization problem in this case. The supplier solves $\frac{a-p_0[t]}{t-c} = \xi[t, p_0[t]]$ as given in (9). We perform a bisection search to find (p_2, t_2). The solution is found to be $(t_2, p_2) = (t_2, p_0[t_2]) = (8.974, 11.009)$, with buyer's net profit $\pi_B[p_2] = \pi_B[11.009] = 6075.747$ as opposed to her best possible profit of $\pi_B[c] = 26165.053$. Meanwhile the supplier attains $\pi_S[8.974] = 13176.616$.

A plot of the change of p_0 with respect to t seems to suggest a linear relationship (constant sensitivity). However, a plot of the change of the sensitivity ξ of market price with respect to t in Figure 3 shows that this is not the case. Between values of $t = c$ and $t = \bar{t}$ there is a change of $\sim$10%. This shows that the buyer's reaction price p_0 becomes increasingly sensitive to the supplier's wholesale price t. This can be explained as follows: As the wholesale price becomes greater, the buyer has to account not only for this increase in prices, but also with the fact that the operational costs become a greater percentage of total costs. The market becomes more and more of a niche market, and market price increases nonlinearly.

Next we consider the buyer-driven case where the buyer is interested in maximizing his net profit function $P_B[\alpha, \beta]$ shown in the contour plot of Figure 4. It is easy to see that $\alpha \approx 1.6$ and $\beta = 0$ at the highest point of the surface. The precise values are $(\alpha^*, \beta^*) = (1.600, 0)$. The reaction wholesale price of the supplier is $t_3^* = 6.799$ and the market price is $p_3^* = p_3 = 10.880$. Even though it is optimal to have $\beta^* = 0$ at α^*, that's not the case for other α values. So if value were fixed to $\alpha = \alpha_F$ due to certain conditions in the channel, one would set β to the unique positive value that corresponds to the β value of the point where the surface $\alpha = \alpha_F$ is tangent to the contour line of $P_B[\alpha, \beta]$. On the other hand, for a fixed R_B (an isoprofit curve in Figure 4) and a fixed α, there exist two possible choices of β, and the buyer should select the one that yields greater profits for the supplier. Finally, we compute optimal β for $\alpha = 1$ as $\beta^* = 3.765$. This is where the buyer is restricted to declaring only a markup (thus *"β-only buyer-driven"*).

Tables 1, 2 and 3 show the profits, prices and sensitivities respectively in different channel structures. The total profit in the coordinated channel structure is the maximum, followed by the buyer-driven structure. This result is due to the fact that our model assumes logistics costs only at the buyer, not at the supplier, and in the buyer-driven case the buyer declares her multiplier (and thus sensitivity) a high value to force the supplier choose a lower wholesale price. The increase in total profit when one goes from supplier-driven to buyer-driven is $\sim 4.3\%$.

Channel Structure	Supplier's Profit	Buyer's Profit	Total Profit
Coordinated	0	26165.053	26165.053
Supplier-driven	13176.616	6075.747	19252.363
Buyer-driven	5020.9396	15053.371	20074.3106
β-only Buyer-driven	6429.454	12111.658	18541.112

Table 1: Profits in different channel structures

Channel Structure	Wholesale Price	Market Price
Coordinated	$t_1 = 5.600$	$p_1 = 9.269$
Supplier-driven	$t_2 = 8.974$	$p_2 = 11.009$
Buyer-driven	$t_3 = 6.800$	$p_3 = 10.880$
β-only Buyer-driven	$t_\beta = 7.317$	$p_\beta = 11.083$

Table 2: Prices in different channel structures

Channel Structure	Sensitivity at the Optimal
Coordinated	0.5112
Supplier-driven	0.531
Buyer-driven	1.600
β-only Buyer-driven	1.000

Table 3: Sensitivity $\xi[t, p_0[t]]$ of market price to wholesale price in different channel structures

We also performed a numerical analysis to observe the impact of changing (m, k) and (m, r) on prices and sensitivity of market price. For the supplier-driven model we used `Mathematica` and searched for t_2 by a bisection routine. For the buyer-driven model we used online `SNOPT` nonlinear optimization solver at NEOS server [**?**], and expressed the model in `AMPL` language.

Figure 5 shows the change of p_2 with changing values of the market demand multiplier m and the ordering cost k. One observation is that as m increases, p_2 *decreases*, which suggests economies of scale in the supply chain due to reduced ordering cost per unit. This tells that as the market size increases, decreasing the price of the product is more beneficial for the buyer (since smaller market price takes place only when the wholesale price is smaller, we infer that the wholesale price also decreases with larger m). Similar economies of scale results helping all players. This pattern is due to the fact that the ordering cost does not increase proportionally as demand increases. Also, the impact of increased demand is much more pronounced in smaller values of m (up to a point). For a fixed m value, p_2 (and thus t_2) *increase* with increasing values of the ordering cost k. However, the increase in p_2 becomes smaller at higher values of k and m. Meanwhile, it's interesting to note that this economies of scale take place only after a certain market size, and higher k requires greater demands for economies of scale to come into effect.

A similar analysis of change with respect to (m, r) shows that for a fixed m value, p_2 (and thus t_2) *increase* with increasing values of the ordering cost r. However, the increase in p_2 becomes smaller at higher values of r and m.

The same patterns for the price (p_3) is observed in the buyer-driven case and the sensitivities ($\xi[t_2, p_2]$ and α^*) in supplier and buyer-driven cases.

6 Comparison of Supplier-driven and β-only Buyer-driven Channels

In this section we compare the supplier-driven channel to β-only buyer-driven channel. We show that there exist problem instances where the buyer may prefer the supplier-driven to β-only buyer-driven channel and similarly problem instances where the supplier may prefer the β-only buyer-driven channel to supplier-driven. For notational simplicity, we show these results only for the classic bilateral monopoly (BM) model, where operational costs are zero ($k = 0, r = 0$).

In BM, by setting $\frac{\partial \pi_B}{\partial p} = 0$ we find that the buyer selects $p^{(0)} = (a+t)/2$ as the market price. When $p^{(0)}$ is substituted, the supplier's profit function becomes $\pi_S = (t-c)m(a-t)/2$. By setting $\frac{\partial \pi_S}{\partial p} = 0$ we find that the supplier selects $t^{(0)} = (a+c)/2$ as the wholesale price, and the buyer sets $p^{(0)} = (a+t)/2 = (3a+c)/4$ as the market price. The optimal profits can be easily derived by substitution as $\pi_S^{(0)} = m(a-c)^2/8$ and $\pi_B^{(0)} = m(a-c)^2/16$.

For a fixed α, the optimal β can be derived by setting $\frac{\partial \pi_B^{(\alpha,\beta)}}{\partial \beta} = 0$ and solving for β. The two roots are:

$$\beta = \{a + c\alpha^2 - \alpha\sqrt{c(2a - c + 2c\alpha + c\alpha^2)}, \tag{12}$$

$$a + c\alpha^2 + \alpha\sqrt{c(2a - c + 2c\alpha + c\alpha^2)}\} \tag{13}$$

β should be less than $a - c$, since a larger β would cause zero market demand. Since the second root is always greater than a (and thus $a - c$), we use only the first root.

In realistic settings, we can normalize to a fixed $\alpha = 1$ (i.e. the multiplier-markup pair $(1, \beta)$). The supplier selects $t^{(\beta)} = (a - \beta)/2$, and the buyer sets the market price to $p^{(\beta)} = (a + \beta)/2$. When these are substituted into $\pi_B^{(\beta)}$, one obtains $\beta = (a + c) - \sqrt{2c}\sqrt{a + c}$. Substituting this back into the price and profit functions, we obtain:

$$t^{(\beta)} = \sqrt{c(a + c)/2}, \tag{14}$$

$$p^{(\beta)} = (a + c) - \sqrt{c(a + c)/2}, \tag{15}$$

$$\pi_S^{(\beta)} = m\left(-2c + \sqrt{2c(a + c)}\right)/4, \tag{16}$$

$$\pi_B^{(\beta)} = -m\left(-2c + \sqrt{2c(a + c)}\right)\left(-(a + c) + \sqrt{2c(a + c)}\right)/2 \tag{17}$$

The pricing scheme that involves only β is not the most advantageous scheme for the buyer. The buyer would always prefer a pricing scheme that involves a multiplier when possible. Similarly, the supplier would always prefer supplier-driven to the buyer-driven (where multiplier is allowed).

An interesting question is whether the buyer would ever be willing to give up the β-only pricing scheme and prefer a *supplier-driven*. The next theorem answers this question, and is obtained by comparing π_B under β-only and supplier-driven models:

Theorem 8 *In BM, the buyer would prefer supplier-driven setting to β-only buyer-driven when*

$$(a - c)^2 - 8\left(-2c + \sqrt{2c(a + c)}\right)\left(-(a + c) + \sqrt{2c(a + c)}\right) > 0 \tag{18}$$

Proof:

$$\pi_B^{(0)} > \pi_B^{(\beta)} \tag{19}$$

$$m(a - c)^2/16 - m\left(-2c + \sqrt{2c(a + c)}\right)\left(-(a + c) + \sqrt{2c(a + c)}\right)/2 > 0 \tag{20}$$

$$(a - c)^2 - 8\left(-2c + \sqrt{2c(a + c)}\right)\left(-(a + c) + \sqrt{2c(a + c)}\right) > 0 \tag{21}$$

□

The instances where (18) holds are characterized by very high a compared to c. If we similarly consider the supplier's preferences:

Theorem 9 *In BM, the supplier would prefer supplier-driven setting to β-only buyer-driven when*

$$(a-c)^2 + 4c - 2\sqrt{2c(a+c)} > 0 \tag{22}$$

Proof:

$$\begin{aligned} \pi_S^{(0)} &> \pi_S^{(\beta)} & (23)\\ m(a-c)^2/8 - m\left(-2c + \sqrt{2c(a+c)}\right)/4 &> 0 & (24)\\ (a-c)^2 + 4c - 2\sqrt{2c(a+c)} &> 0 & (25) \end{aligned}$$

□

The instances where (22) holds are characterized by a being greater than c. Theorems 8 and 9 tell us that there does not exist a region where both parties prefer the same case. Thus what determines which case will occur is the distribution of the bargaining power in the channel.

7 Conclusions

In this paper we have explored the impact of power structure on price, sensitivity of market price and profits in a two-stage supply chain. Following analysis of the buyer's decision problem for a given wholesale price, we analyzed both the supplier-driven and buyer-driven cases.

We showed that if the buyer uses a linear form of a price increase that it is optimal for the buyer to set the markup to zero and use only a multiplier. We also observed that the market price and its sensitivity with respect to the wholesale price becomes greater in the supplier-driven case compared to a channel where operational costs at the buyer are ignored. We found that the sensitivity of the market price increases non-linearly as the wholesale price increases. In addition, we observed that marginal impacts of increasing shipment cost and carrying charge (interest rate) on prices and profits are decreasing in both cases. Finally, we showed that there are cases when a buyer will actually prefer a supplier-drvien channel to a buyer-driven one and where a supplier will prefer a buyer-driven channel to a supplier-driven one.

Although, as we discussed earlier, there are many cases where the assumption of full information is applicable, there are many cases where information assymetries arise. In this case additional strategic considerations must be incorporated into the analyses.

Appendix

A Proofs

A.1 Theorem 2

Proof: We are interested in characterizing the three roots of $\Phi(p) = 0$ given that they exist. The case where a real cubic equation has all real and distinct roots is known as the *"irreducible case"*, and holds when its discriminant

$$\Delta = 18\Phi_0\Phi_1\Phi_2 - 4\Phi_0\Phi_2^3 + \Phi_1^2\Phi_2^2 - 4\Phi_1^3 - 27\Phi_0^2 \tag{26}$$

is positive. (Dickson (1939) [**?**], p48-50) describe a *"trigonometric solution"* method for this case.

If we let $p_0 = argmax_{\{p=p_{0,1},p_{0,2},p_{0,3}\}}\pi_B[p]$ then $p_0 < \tilde{p}$. This follows from Theorem 1: $\pi_B[p]$ is concave when $p < \tilde{p}$ and has a maximum in the interval $(t, \tilde{p})$. $\pi_B[p]$ is convex when $p > \tilde{p}$ and has a minimum in the interval $(\tilde{p}, a)$. Thus the root with the maximum net profit value satisfies $p_0 < \tilde{p}$.

We derived the polynomial equation $\Phi(p) = 0$ starting with $(p - w/2) = k/(2q)$.

Note that taking the square of both sides, we introduce a new solution to the equation, namely the p value such that $|p - w/2| = p - w/2$, even though $p - w/2$ is restricted to positive values. So we have a new p value, which is neither an extreme nor an inflection point, that satisfies $\Phi(p) = 0$. This p value also satisfies $p - w/2 < 0$ and thus $p < w/2$. Since the other roots satisfy $p - w/2 > 0$ and thus $p > w/2$, the mentioned p value is the minimum $p_{0,min}$ of the three roots $\{p_{0,1}, p_{0,2}, p_{0,3}\}$ of $\Phi(p) = 0$. We can also deduce that the maximum of $\pi_B[p]$ is achieved at $p_0 \in (w/2, \tilde{p})$, and is the smaller of the two roots that satisfy $p > w/2$, that is, $p_0 = \min\{p_{0,1}, p_{0,2}, p_{0,3}\} \backslash \{p_{0,min}\}$. $\square$

A.2 Theorem 3

Proof: Equation (5) gives the relation between a given t and the supplier's optimal price p_0. Taking the partial derivative of both sides with respect to t we obtain:

$$
\begin{aligned}
3p^2\frac{\partial p}{\partial t}+\frac{\partial \Phi_2}{\partial t}p^2+2p\frac{\partial p}{\partial t}\Phi_2+\frac{\partial \Phi_1}{\partial t}p+\Phi_1\frac{\partial p}{\partial t}+\frac{\partial \Phi_0}{\partial t} &= 0\\
\left(3p^2+2\Phi_2 p+\Phi_1\right)\frac{\partial p}{\partial t}- & \\
(1+r\tau)p^2+ & \\
((1+r\tau)(4a+w)+(1+r\tau)w)\,p/4- & \\
a(1+r\tau)(2w)/4+kr/(8m) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\xi_1[t,p]\frac{\partial p}{\partial t}- & \\
\left((1+r\tau)p^2-(1+r\tau)(2a+w)p/2+a(1+r\tau)w/2-kr/(8m)\right) &= 0\\
\xi_1[t,p]\frac{\partial p}{\partial t}- & \\
\left((1+r\tau)\left(p^2-(2a+w)p/2+aw/2\right)-kr/(8m)\right) &= 0\\
\xi_1[t,p]\frac{\partial p}{\partial t}-\xi_0[t,p] &= 0
\end{aligned}
$$

From the last line we have $\frac{\partial p}{\partial t}=\frac{\xi_0[t,p]}{\xi_1[t,p]}$. When we substitute $p=p_0[t]$ we have $\frac{\partial p_0[t]}{\partial t}=\frac{\xi_0[t,p_0[t]]}{\xi_1[t,p_0[t]]}$. $\square$

A.3 Theorem 4

Proof: Our proof has two steps: First we show that $F[w,p]=\xi_0[t,p]-\xi_1[t,p]/2<0$. Next we will show that $\xi_1[t,p]/2<0$, which enables us to state that $\xi[t,p]=\frac{\xi_0[t,p]}{\xi_1[t,p]}$.

¿From (4), we can obtain $w=2p-\Delta$, where $\Delta=k/2q>0$. We substitute this into $F[w,p]$ at the very first step to simplify our analysis:

$$
\begin{aligned}
F[w,p] &= F[2p-\Delta,p]\\
&= \xi_0[t,p]-\frac{1}{2}\xi_1[t,p]\\
&= (-kr/m-\Delta^2-4ar\Delta\tau+4pr\Delta\tau)/8\\
&= (-kr/m-\Delta^2-4r\Delta\tau(a-p))/8
\end{aligned}
$$

The last line is composed of a numerator with three negative components, thus $F[w,p]<0$.

Since $F[w,p]=\xi_0[t,p]-\xi_1[t,p]/2<0$, we can just take the second term to the right hand side and divide both sides by $\xi_1[t,p]$. If $\xi_1[t,p]<0$ then $\xi[t,p]>1/2$. The last and necessary step is showing that $\xi_1[t,p] < 0$, so that the inequality changes signs when division by $\xi_1[t,p]$ takes place.

A careful inspection of (5), (6) and (8) reveals that $\xi_1[t,p] = \frac{d\Phi(p)}{dt}$. From Theorem 2b, for a given t value, p_0 is the second largest root of $\Phi(p) = 0$. Since $\Phi(w/2) > 0$, $\Phi(p)$ decreases from the positive value of $\Phi(w/2)$ at $p = w/2$ to the value of $\Phi(p_0) = 0$ at $p = p_0$. More formally,

$$\xi_1[t,p_0] = \frac{d\Phi(p_0)}{dt} < 0$$

Thus when we divide both sides of the inequality $\xi_0[t,p] < \xi_1[t,p]/2$ by $\xi_1[t,p]$, the sign changes direction. $\square$

A.4 Theorem 5

Proof:

(9) tells us that $\frac{a-p_2}{t_2-c} = \xi[t, p_0[t]]$

We are interested in showing that this indeed gives us the global optimum, by showing the concavity of $\pi_S[t]$. Thus, we are interested in showing that

$$\frac{\partial^2 \pi_S[t]}{\partial t^2} = -2\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial t^2}(t-c) < 0$$

We can use (9) in the above expression when substituting $\frac{\partial p}{\partial t}$ and evaluating $\frac{\partial^2 p}{\partial t^2}$:

$$\begin{aligned}
-2\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial t^2}(t-c) &= \\
-2\frac{a-p}{t-c} - \frac{\frac{\partial p}{\partial t}(t-c) - (a-p)}{(t-c)^2}(t-c) &= \\
\frac{-2(a-p) - \frac{a-p}{t-c}(t-c) + (a-p)}{(t-c)} &= \\
-2\frac{(a-p)}{(t-c)} &< 0
\end{aligned}$$

where the last inequality follows from $(a-p)$ and $(t-c)$ being positive. $\square$

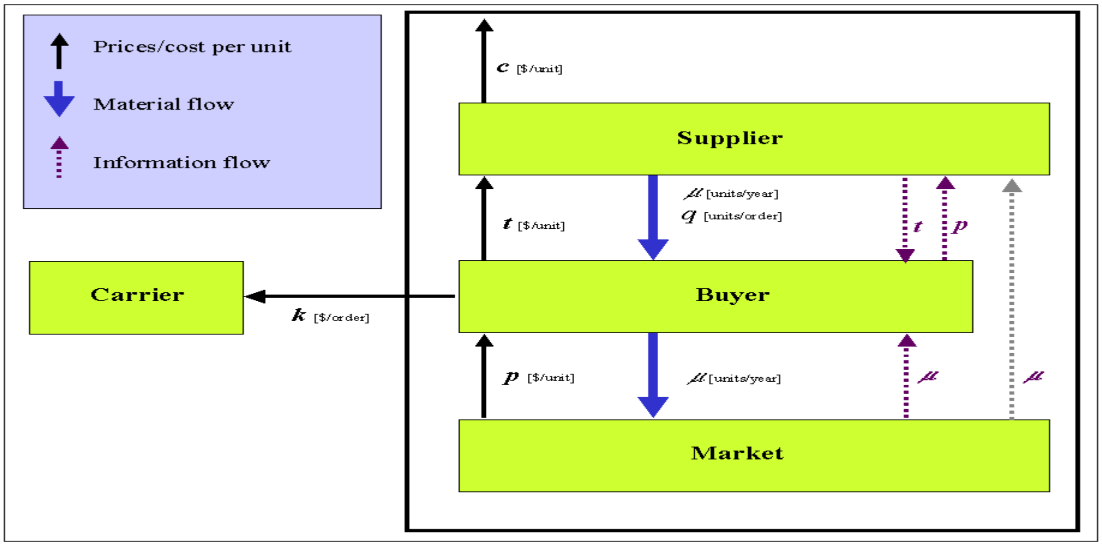

Figure 1: The supply chain channel

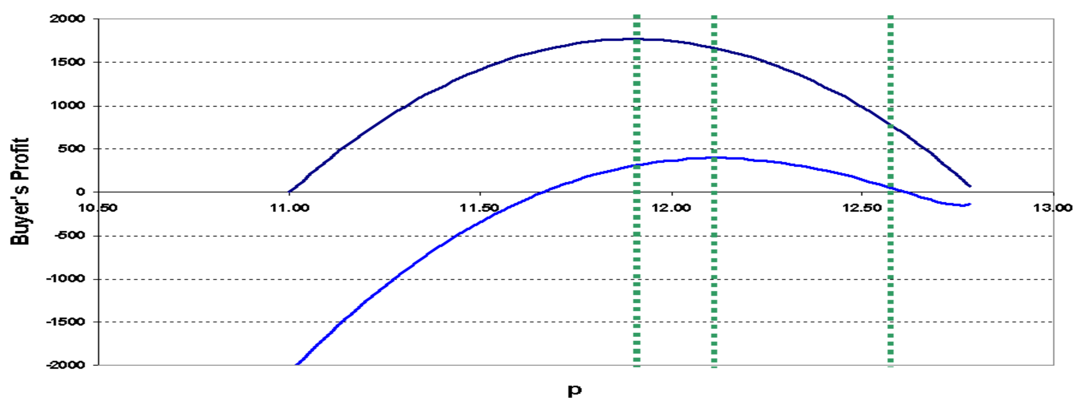

Figure 2: Profits/Costs for the buyer when $t = 11$

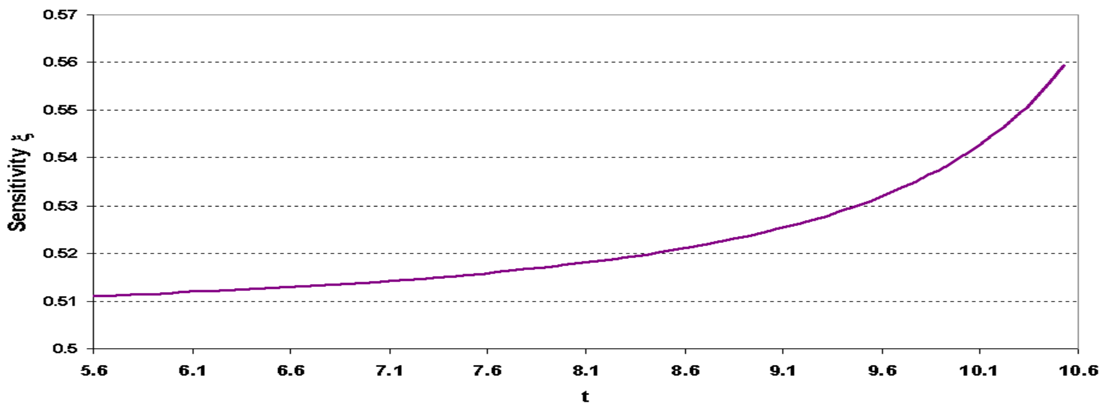

Figure 3: Change of the sensitivity (ξ) of p with respect to t in the supplier-driven case

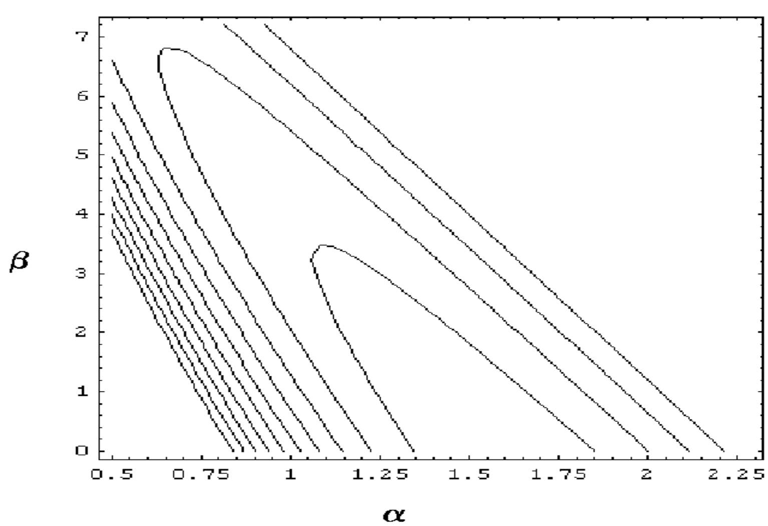

Figure 4: $P_B[\alpha, \beta]$, contour plot

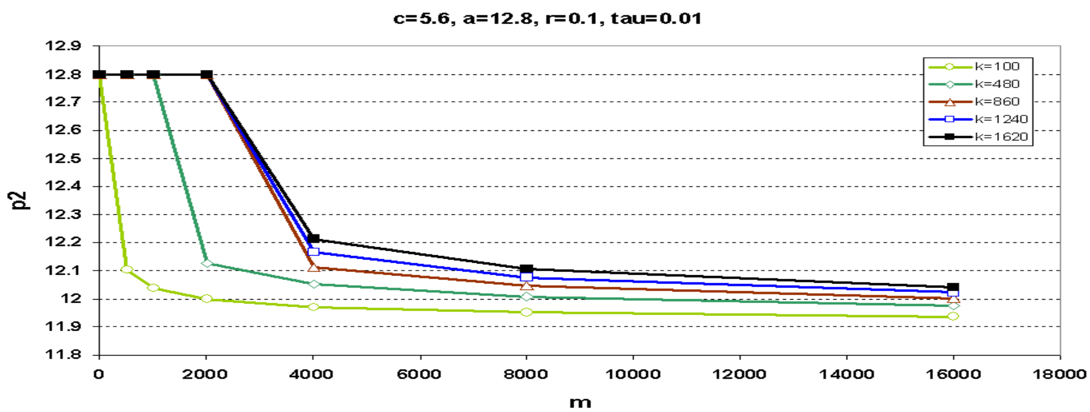

Figure 5: Change of p_2 with respect to m and k in the supplier-driven case